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FINAL REPORT.

International Conference:

Applied Probability Models for
Complex Stochastic Systems,

held at

Jane S. McKimmon Center
North Carolina State University
Raleigh, NC 27650

January 8-9, 1981

conference organizer: S. Stidham, Jr.

sponsored by: Army Research Office (DAAG29-81-M-0043)
P. O. Box 12211
Research Triangle Park, NC 27709

Graduate Program in Operations Research
North Carolina State University
Raleigh, NC 27650

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A conference on "Applied Probability Models for Complex Stochastic Systems" was held at Jane S. McKimmon Center, North Carolina State University, on January 8-9, 1981. The conference was organized by Dr. Shaler Stidham, Jr., Professor in the Department of Industrial Engineering and Graduate Program in Operations Research, North Carolina State University.

The purpose of the conference was to bring together several prominent mathematicians and engineers from Europe and the U.S. with common interests in applied probability models for complex stochastic systems, to interchange ideas with one another and with faculty, students, and researchers at the universities and research organizations in the Research Triangle area.

Primary emphasis was placed on two topics: (i) distribution-free analysis of complex stochastic systems; and (ii) optimal control of complex stochastic systems. This emphasis was motivated by the need for mathematical models to help describe and control such complex stochastic systems as networks of queues, such as arise in computer and communication systems and industrial job shops, and operating systems composed of parts subject to maintenance, failure, and replacement, such as arise in aerospace and military applications. (More detailed discussion of the topics covered and the rationale for choosing them can be found in the proposal for the conference, which is Appendix A to this report.)

The speakers at the conference were:

Dr. David Burman, Bell Laboratories

Prof. Awi Federgruen, Columbia University

Prof. Robert Foley, Virginia Polytechnic Institute

Prof. Peter Franken, Humboldt University, Berlin (visiting Virginia Polytechnic Institute)

Dr. Daniel P. Heyman, Bell Laboratories

Prof. Frank Kelly, Cambridge University

Prof. S. S. Nair, St. Augustine's College, Raleigh

Prof. N. U. Prabhu, Cornell University

Prof. Tomasz Rolski, Wrocław University, Poland

Prof. Rolf Schassberger, Technical University, Berlin

Dr. Richard F. Serfozo, Bell Laboratories

Dr. Burton Simon, Bell Laboratories

Dr. D. R. Smith, Bell Laboratories

Prof. Henk Tijms, Free University, Amsterdam

Prof. Jean Walrand, Cornell University

Prof. Jaap Wessels, Eindhoven University, The Netherlands

A copy of the program for the meeting, including titles of the talks, is included in Appendix B to this report. Detailed abstracts and, in some cases, complete papers for the talks are included in Appendix C.

We wish to express our gratitude to each of the speakers for the time and effort that went into their talks, which contributed to the success of the meeting. Thanks are also due to Prof. Ralph Disney, who attended the meeting, and to the other organizers of the TIMS meeting on "Applied Probability-Computer Science: The Interface", held January 5-7, 1981, at Florida Atlantic University, Boca Raton, Florida. Most of the speakers at our conference also attended that meeting, without which it would not have been possible to bring together so many distinguished researchers in Raleigh. We are especially grateful to Prof. Disney for his encouragement, helpful suggestions, and cooperation, particularly in making it possible for Prof. Franken to attend.

In addition to the speakers, there were about 30-35 participants from the Research Triangle area and elsewhere. These included the following persons:

Prof. John W. Bishir, North Carolina State University

Prof. Halim Dogrusoz, North Carolina State University

Prof. S. E. Elmaghraby, North Carolina State University

Prof. George S. Fishman, University of North Carolina, Chapel Hill

Prof. Arnold Greenland, George Mason University

Dr. Robert L. Launer, Army Research Office

Prof. Ross Leadbetter, University of North Carolina, Chapel Hill

Prof. Louis Moore, University of North Carolina, Chapel Hill

Prof. Arne Nilsson, North Carolina State University

Prof. Henry L. W. Nuttle, North Carolina State University

Prof. Walter L. Smith, University of North Carolina, Chapel Hill

Prof. M. Venkatesen, Case Western Reserve University

Dr. Gerald Andersen, Army Research and Development Command

We would like to thank the Army Research Office for providing the support which made this conference possible. Special thanks should go to Dr. Robert L. Launer of ARO, for his personal support and his efforts to expedite the proposal on unusually short notice.

Finally, our sincere thanks to Prof. Salah Elmaghraby, Director of the Graduate Program in Operations Research, for his encouragement which led to the idea of holding the conference, for his unselfish commitment of time and advice to help make it a success, and the financial support provided by the Operations Research Program.

<u>APPENDIX A</u>

PROPOSAL FOR CONFERENCE

A Proposal for a Conference on

APPLIED PROBABILITY MODELS FOR COMPLEX STOCHASTIC SYSTEMS

Subject of Proposal

A conference on "Applied Probability Models for Complex Stochastic Systems," to be held at North Carolina State University, January 8-10, 1981, with 5-10 invited speakers (prominent researchers from Europe and the United States) and 10-20 additional participants (faculty and students from the Research Triangle universities and applied researchers from Research Triangle Park and local industry).

Conference Theme and Topics

The subject of the conference will be applied probability models for complex stochastic systems, particularly multi-dimensional systems of interrelated components. Examples are networks of queues, such as arise in computer and communication systems and industrial jobs shops, and operating systems composed of parts subject to maintenance, failure, and repair or replacement, such as arise in aerospace and military applications. These topics are currently of great interest both to applied probabilists, who are developing descriptive and normative models for such systems, and to users and potential users of these models in industry and the military.

Rather than consider a broad range of possible models, we intend to place primary emphasis on two topics:

(i) distribution-free analysis of complex stochastic systems; and

(ii) optimal control of complex stochastic systems.

Under the first topic, we shall focus attention on such subjects as conservation relations, the insensitivity phenomenon, bounds, approximations, and monotonicity of systems. Investigation of these subjects will be based primarily on the theory of regenerative processes and the more general random marked point processes (RMPP), sample-path (operational) analysis, and stochastic ordering. Under the second topic, we shall consider the structure of optimal control policies and efficient computational algorithms, based primarily on the theory of Markov decision processes (MDP) and the techniques of dynamic programming.

Rationale for Conference

There are several reasons for proposing a conference on these topics at this time.

First, except for simple exponential models (such as the Jackson network), multi-dimensional models in queueing and reliability theory are notoriously difficult to solve by conventional techniques, such as Markov-process theory. The theory of RMPP has provided a tool for studying such systems, without the requirement for exponential, or even independent, interarrival, service, failure, or repair time distributions. Together with the concept of Palm probabilities, this theory has facilitated a unified approach in a distribution-free setting to such problems as the relation between customer and time averages in queues, the relation between a continuous-parameter stochastic process and an imbedded discrete-parameter process, and the insensitivity of equilibrium distributions in queueing networks. Parallel, but so far more limited results, have been achieved using sample-path analysis. Using some of these results, together with the concept of stochastic ordering, researchers in applied probability have been able to approximate complex stochastic systems by simple systems, with tight upper and lower bounds for such measures as expected queue lengths and waiting times.

The theory of MDP, while not extending the range of models that can be solved analytically, has focussed attention on optimization of stochastic systems, often revealing the structure of optimal control policies and hence indicating the direction in which descriptive modelling should proceed. In addition, recent computational developments in dynamic programming theory have increased the potential scope for efficient numerical computation of optimal control policies, even for multi-dimensional systems.

A second reason for choosing these topics is that research in each has been developing in recent years at a rapid pace in Europe (primarily West and East Germany and the Netherlands). As a consequence, applied probabilists in the U.S. are perhaps not as aware of the latest developments as they might otherwise be.

Finally, at the time of the proposed conference, several researchers with interests in these topics from Europe and the U.S. will be returning from the conference on "Applied Probability-Computer Science, The Interface," sponsored by the ORSA/TIMS Special Interest Group/College on Applied Probability, to be held at Florida Atlantic University, Boca Raton, Florida, January 5-7, 1981. It will be convenient for some of them to stop in Raleigh after that conference. (It was the fact that several of the proposed participants contacted us and indicated a desire to visit North Carolina State University on the way home from the Florida conference that originally provided the impetus for organizing a conference in Raleigh.)

While the topics proposed for this conference will also form part of the agenda at the Florida meeting, the two conferences should be viewed as complementary rather than competitive in scope and purpose. The theme of the Florida meeting - the application of applied probability models to computer systems - will be one of several potential applications of the theoretical topics in the proposed conference. On the other hand, with its focus on

distribution-free analysis and control models for complex systems, the proposed conference promises to be more narrow in scope. Moreover, we expect to have a much smaller number of speakers (5-10 vs. 40-50) and a more homogeneous group of participants, with more opportunities to pursue the selected theoretical topics in depth.

Benefits of Conference

The conference will take place over a period of 2 1/2 - 3 days and will be organized around 5-10 invited papers, with ample time for discussion of the papers and informal communication among the participants. We hope for two main benefits to come from the conference. The first will be the mutual stimulation from bringing together a small group of applied probabilists of similar interests, each of whom has contributed significantly to the development of models for complex stochastic systems. The second will be the dissemination of knowledge from the invited speakers to the applied probability community in the Research Triangle area, including faculty and students at the area universities as well as applied researchers from Research Triangle Park and local industry. There is an active group of faculty and students at North Carolina State University who have been contributing substantially in recent years to research on applied probability, particularly optimal control of queueing systems and modelling and numerical analysis of computer and communication systems. It is hoped that the conference will help bring recognition and added stimulus to the efforts of this group.

Possible Participants

A tentative list of invited speakers follows. (Those marked by a * have been contacted and have indicated a desire to come.)

*Ralph Disney, Virginia Polytechnic Institute and State University

*Peter Franken, Humboldt University, Berlin

Daniel P. Heyman, Bell Telephone Laboratories

Frank Kelly, Cambridge University

*Dieter Konig, Bergakademie Freiberg

*Austin Lemoine, Systems Control, Inc., and Stanford University

*Teunis Ott, Bell Telephone Laboratories

*N. U. Prabhu, Cornell University

*Tomasz Rolski, Wrocław University

Rolf Schassberger, Technical University, Berlin

*Richard Serfozo, Bell Telephone Laboratories

*Henk Tijms, Free University, Amsterdam

*Jaap Wessels, Eindhoven University, The Netherlands

Ward Whitt, Bell Telephone Laboratories

The following is a partial list of researchers from the Research Triangle area who might also participate.

B. B. Bhattacharyya, North Carolina State University

John W. Bisher, North Carolina State University

W. W. Chou, North Carolina State University

Halim Dogrusoz, North Carolina State University

George Fishman, University of North Carolina, Chapel Hill

R. Leadbetter, University of North Carolina, Chapel Hill

Arne Nilsson, North Carolina State University

Walter Smith, University of North Carolina, Chapel Hill

William R. Stewart, North Carolina State University

Other interested participants will also be welcomed.

BUDGET

January 8-10, 1981

	Requested from ARO	From NCSU and Other Sources
*1) Invited Speakers: travel and living expenses @ $400/person	$ 2400	$ 2800
2) Preparation of Papers	1000	--
3) Local Participants: travel and meal expenses for 10 persons	500	--
4) Facilities: room charges, etc., at McKimmon Center	200	--
5) Miscellaneous: postage, telephone, etc.	250	--
*6) Travel Expenses: for Dr. F. P. Kelly Round trip airfare England-U.S.	600	--
TOTAL	$ 4950	$ 2800

*All the invited participants, with the exception of Dr. Kelly, expect to receive support from other sources to attend the Florida TIMS/ORSA meeting ("Applied Probability - Computer Science, The Interface," January 5-7, 1981). Consequently we are requesting only incremental travel expenses to and from Boca Raton, Florida, for those invited speakers who need such support. In the case of Dr. Kelly, however, we are requesting support for round-trip travel from England to the U.S. as well, because we have been informed by Dr. Teunis Ott, Organizing Chairman of the Florida meeting, that without such support he will not be able to attend either meeting.

NOTE: No money from ARO will be used to invite speakers from Eastern Block countries.

Civil Rights Compliance Statement

"No person will, on the grounds of race, color or national origin be excluded from participation, be denied the benefits of, or be subjected to discrimination under this program."

Affirmative Action Statement

"NCSU does not discriminate on the basis of sex, race or handicap and is an affirmative action/equal employment institution."

APPENDIX B

CONFERENCE PROGRAM

REVISED PROGRAM

Conference on Applied Probability Models

for Complex Stochastic Systems

January 8-9, 1981

McKimmon Conference Center
North Carolina State University
Raleigh, NC

Schedule (subject to change)

(Shuttle bus will leave Mission Valley Inn at 8:30 each morning, return from McKimmon Center at 5:30 each evening.)

Thursday, January 8

8:45 - 9:00	Registration
9:00 - 9:15	Opening remarks
9:15 - 10:00	R.F. Serfozo, "Compound Point Processes and Rare Events"
10:00 - 10:45	Rolf Schassberger, "Networks of Queues in Discrete Time"
10:45 - 11:00	Coffee break
11:00 - 11:45	Frank Kelly, "Networks with Partial Balance"
11:45 - 12:30	Jean Walrand, "Filtering Theory and Networks of Queues"
12:30 - 2:00	Lunch*
2:00 - 2:45	David Burman, and D.R. Smith, "A Light-Traffic Theorem for Multi-Server Queues"
2:45 - 3:30	N.U. Prabhu, "Comparison of Random Walks, with Applications to Queues"
3:30 - 3:45	Coffee break
3:45 - 4:30	Robert Foley, "Compartmental Models and Marked Point Processes"

4:30 - 5:15 S. S. Nair, "A Carrier Shuttling between Two Terminals"

6:30 Cocktails and dinner (Balentine's Restaurant, Cameron Village)**

Friday, January 9

9:00 - 9:15	Announcements
9:15 - 10:00	Henk Tijms, "Stochastic Control for Specially Structured Optimization Problems"
10:00 - 10:45	Awi Federgruen, "Approximations of Dynamic, Multi-Location Production and Inventory Problems"
10:45 - 11:00	Coffee break
11:00 - 11:45	Jaap Wessels, "Using the Problem Structure in Large Markov Decision Processes"
11:45 - 12:30	Peter Franken, Topic to be announced.
12:30 - 2:00	Lunch*
2:00 - 2:45	Daniel Heyman, "Conservation Laws for Queues"
2:45 - 3:30	Tomasz Rolski, "Queues with Non-Stationary Input Stream: Ross's Conjecture"
3:30 - 3:45	Coffee break
3:45 - 4:30	Burton Simon, "Priority Queues with Feedback"

* Included in registration fee of $20.00 (or $10.00 for one day). Spaces are limited. Registration fee for conference only (no lunch): $4.00 per day (no charge for NCSU students).

** Dinner reservations: $10.00/person. Spaces are limited.

For further information contact:

Professor Shaler Stidham
North Carolina State University
Department of Industrial Engineering
Box 5511, Raleigh, NC 27650
(919)733-2362

APPENDIX C

ABSTRACTS OF TALKS

Abstract of

Compound Point Processes and Rare Events

by

Richard Serfozo
Bell Laboratories, Holmdel N.J.

It is well known that the binomial distribution for rare events converges to (and hence can be approximated by) a Poisson distribution. I will show how this convergence result for distributions of rare events extends to point processes. First, a "compound" point process for events with attendant random variables will be defined using Laplace functionals. Then several theorems will be presented that characterize the convergence of these compound processes as their defining parameters tend to values indicative of rare events. Applications to partitioning and merging of streams of customers in networks will also be discussed.

Networks of Queues in Discrete Time

by

Rolf Schassberger

Technical University

Berlin

ABSTRACT

The networks of queues described and analyzed in this talk consist of so-called doubly stochastic servers. When in isolation such a server operates in discrete time as follows: the time is sliced into quanta of equal length; during each quantum at most one job may arrive at the server, the probability for an arrival being λ, $\lambda \in (0,1)$; an arrival is of type r, $r \in R$, with probability α_r, where R is a countable set and $\sum_{r \in R} \alpha_r = 1$; an arrival of type r carries a request of ℓ_r quanta of service time, $\ell_r = 1,2,\ldots$; the server hands out one quantum of service to each job immediately upon arrival and thereafter operates under a so-called doubly stochastic discipline; a complete description of these disciplines being fairly involved, it may be sufficient here to state that they include round-robin, last-in-first-served-preemptive-resume, and random-service scheduling, all with the additional feature of queue-size dependent service capacity; upon completion of its request a job departs from the server.

It is clear from this description that the doubly stochastic server is a rather important model. For instance, under certain specifications of the parameters, it becomes essentially a discretized M/G/1 queue under round-robin discipline. The state of the doubly stochastic server, suitably defined and observed at the successive beginnings of the time slices, fluctuates as a discrete-state Markov chain. The steady-state distribution of this chain is

found and turns out to be of a simple product-form. A similar result for the M/G/1 round-robin queue is not available. The average response time of a job of type r is found as well and turns out to be linear in ℓ_r.

Now it is shown how doubly stochastic servers can be linked together to form networks whose modelling power is similar to the well-known networks studied by Baskett, Chandy, Muntz, and Palacios, and by Kelly. Furthermore it is shown, for these networks, that a steady-state law of product-form and a law for the average response time of a job along a given path can be easily derived. It is concluded that the study of these networks should be at least as useful as that of their well-known counterparts.

Partial balance

by

F.P. Kelly

Koenig, Matthes and Nawrotski (1967) have presented a detailed analysis of the interrelationship between insensitivity and partial balance within a framework later christened by Schassberger (1977) a generalized semi-Markov scheme. Here we show that the concept of partial balance can be defined for an unstructured Markov process and is then equivalent to various other properties of the process defined in terms of truncation, time reversal, speed changes and observation at entrance and exit times (Kelly, 1979). Finally, we show how the relationship partial balance implies between the invariant measure of a truncated process and the invariant measures of jump chains observed at entrance and exit times can be used as the basis for a sample path construction of a stochastic process exhibiting insensitivity. This approach to insensitivity avoids the requirement that the successive lifetimes involved in the definition of the process be generated by a stationary mechanism.

References

Kelly, F.P. (1979) Reversibility and Stochastic Networks, Wiley, New York.

Koenig, D. Matthes, K., and Nawrotzki, K. (1967) Verallgemeinerungen der Erlangschen und Engsetschen Formeln (Eine Methode in der Bedienungstheorie), Akademie-Verlag, Berlin.

Schassberger, R. (1977) Insensitivity of steady-state distributions of generalized semi-Markov processes, Part I. Ann. Prob. 5, 87-99.

Filtering Theory and Networks of Queues

Jean Walrand, Cornell University

Abstract

The basic idea behind the filtering approach to the analysis of networks of queues is to "calculate" the conditional distribution ξ_t of the state of the network given the past $(N_s, 0 \leq s \leq t)$ of an observed set of flows. The filtering theory leads to the system of (nonlinear) differential equations with jumps solved by ξ_t. A related system of linear equations solved by the un-normalized version of ξ_t can also be derived.

The filtering approach provides a systematic way of making rigorous some otherwise delicate limiting arguments. Some extensions of the arrival theorem in networks of quasi-reversible node constitute an example of those applications. The method also leads to useful necessary conditions for flows in a network to be Poisson.

Using those techniques we could derive the characterization of Poisson flows in single class open networks of quasi-reversible nodes. The result is the intuitive one: A flow on a given link is Poisson if and only if the link is not part of a loop.

Similar ideas can also be used to obtain the conditions under which two given flows in a network have the same law. This is a more difficult problem than the verification of the Poisson character of a flow. Only a few non-trivial examples have been analyzed so far.

Some surprising equivalence results for the interarrival time distributions of flows in networks can be derived by combining methods from filtering theory and first step analysis in Markov chains.

The talk concludes with a remark on a probabilistic argument for the product form.

A LIGHT-TRAFFIC THEOREM FOR MULTI-SERVER QUEUES

by

David Y. Burman
and
Donald R. Smith

Bell Laboratories
Holmdel, New Jersey 07733

ABSTRACT

Several approximations for the expected delay in an M/G/c queue depend on both its light- and heavy-traffic behavior. Although the required heavy-traffic result has been proved, the light-traffic result has only been conjectured by Boxma, Cohen, and Huffels. We prove this conjecture when the service is of phase-type; intuitively, any M/G/c queue can be arbitrarily closely approximated by such a system. In particular, as the traffic goes to zero, we show that the probability of delay depends only on the mean of the service-time distributions and that the delay (when positive) converges in distribution to the minimum of c independent equilibrium-excess service-times. This result justifies an efficient computational approach to obtain numerical results for M/G/c queues and provides useful methodology for the approximation of other complicated stochastic systems.

Compartmental Models and Marked Point Processes

by

R. D. Foley
Industrial Engineering and Operations Research Department
Virginia Polytechnic Institute and State University
Blacksburg, VA 24061

A compartmental model can be thought of as a system of m locations or compartments. Particles may enter the system, migrate from one compartment to another, or leave the system. The number of particles in each compartment is the usual quantity of interest. For example, consider the United States as the system, each state a compartment, and each person a particle. As people are born, die, emigrate, immigrate, or migrate from one state to another, the populations of the states fluctuate.

Let $0 \leq T_1 \leq T_2 \leq \cdots$ denote the sequence of arrival times of particles to the system. Associated with each particle is a random element, X_n, which describes the behavior of the nth particle in the system. X_n contains the route of the particle and the length of time the particle visits each compartment in its route. Under suitable conditions on the marked point process $\{(X_n,T_n): n > 0\}$, we determine the joint distribution of the number of particles in each compartment at any time t.

The marked point process $\{(X_n,T_n): n > 0\}$ is required to satisfy the following two assumptions:

a. The number of arrivals of particles in the interval $[0,t]$ forms a Poisson process, either stationary or non-stationary;

b. The behavior of any particle is conditionally independent of the behavior of the other particles given the arrival time of the particle. That is,

$$P\{X_n \in B | X_1, T_1, X_2, T_2, \cdots, X_{n-1}, T_{n-1}, T_n\} = P\{X_n \in B | T_n\}.$$

Define $Q_i(t)$ to be the number of particles in compartment i at time t, D(t) to be the number of departures from the system in [0,t], and $K_{ij}(t)$ to be the number of particles that migrate from compartment i to compartment j during [0,t]. Under the assumptions a and b we have shown the following results:

1. $Q_1(t), Q_2(t), \cdots, Q_m(t)$ are independent Poisson random variables;
2. $\{D(t): t \geq 0\}$ is a Poisson process either stationary or non-stationary;
3. $Q_1(t), Q_2(t), \cdots, Q_m(t)$ is independent of $\{D(u): u \leq t\}$;
4. $\{K_{ij}(t): t \geq 0\}$ is a Poisson process iff almost surely each particle migrates from i to j at most once;
5. $\{K_{ij}(t): t \geq 0\}$ and $K_{k\ell}(t): t \geq 0\}$ are independent Poisson processes iff almost surely no particle migrates from i to j and also from k to ℓ.

Result 1 has been proven for tandem compartmental systems in Foley (1981). Jackson & Aspden (1980) has proven result 1 for tandem compartmental systems with a stationary Poisson arrival process and exponential compartmental visit times. Result 1 has been developed for a particular two compartment model in Purdue (1975). Result 1 has been proven for a single compartment model satisfying assumptions a and b in a nice paper by Brown & Ross (1969). Brown and Ross also discuss group arrivals.

Result 2 has been developed for a single compartment system with constant visit times in Ramakrishnan (1980) and for exponentially distributed visit times in Kambo and Bhalaik (1979). Result 2 is related to Mirasol's (1963)

result that the departure process from the $M/GI/\infty$ queue is a stationary Poisson departure process. In Foley (1980), examples are given of queues with non-stationary Poisson arrival processes with stationary Poisson departure processes.

Result 3 is related to the work of Kelly (1979) and Melamed (1979). The property in result 3 appears as a condition in both of these papers in order to obtain the analogue of result 2. Result 3 is quite closely related to the quasi-reversibility condition of Kelly (1979). However it is not exactly the same, since the process may not be stationary. The systems studied in Foley (1980) possess the property in result 3 but are non-stationary.

Results 4 and 5 are the analogues of the results obtained by Melamed (1979) and Walrand and Varaiya (1978) for Jackson queueing networks.

The results obtained for the compartmental models satisfying conditions a and b are strikingly similar to the recent results obtained on Jackson queueing networks and variants thereof by Kelly (1979), Melamed (1979) and Walrand and Varaiya (1978).

References

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Foley, R. D. (1980) Stationary Poisson departure processes from non-stationary queues. Technical Report VTR-8104, IEOR Dept., VPI & SU, Blacksburg, Va.

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Kelly, F. P. (1979) Reversibility and Stochastic Networks. Wiley.

Melamed, B. (1979) On Poisson traffic processes in discrete-state Markovian systems with applications to queueing theory. Advances in Applied Probability 11, #1, 218-239.

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Jackson, R. R. P. and Aspden, P. (1980) A transient solution to the multistage Poisson queueing system with infinite servers. Operations Research 28, #3, 618-622.

Purdue, P. (1975) Stochastic theory of compartments: an open, two compartment, reversible system with independent Poisson arrivals. Bulletin of Mathematical Biology 37, #3, 269-275.

Ramakrishnan, C. S. (1980) A note on the M/D/∞ queue and a paper by Kambo and Bhalaik. Opsearch (to appear)

Walrand, J. and Varaiya, P. (1978) When is a flow in a queueing network Poissonian? Memo No. ERL-M78/59, Electronics Research Laboratory, College of Engineering, University of California, Berkeley.

Stochastic control for specially structured optimization problems

by Henk Tijms,

Vrije Universiteit,
Amsterdam,
Holland.

ABSTRACT. In the last fifteen years there has been a considerable interest in the study of optimal design and control of production and queueing systems, cf. the bibliography by Crabill et al. (1977) and the survey papers by Prabhu and Stidham (1974) and Sobel (1974). On the one hand the literature deals with steady-state analysis of an intuitively reasonable control rule having a simple form and on the other hand a large number of papers are concerned with verifying the optimality of such a simple control rule among a larger class of control rules. However, so far little attention has been paid to the development of computationally tractable algorithms for the numerical solution of the above control problems.

In this lecture we shall present for *structured* applications of *average cost denumerable state* semi-Markov decision problems a computational approach which appeared to be quite succesfull for the applications considered. This approach combines policy-iteration and embedding techniques in such a way that we need only to perform calculations for a *finite* number of states without having truncated the unbounded state space. For a specific application we exploit its *structure* and use an *embedding* technique to develop a tailor-made policy-iteration algorithm which deals only with a finite embedded set of states in any iteration and generates a sequence of improved policies having the desired simple form. In each of the applications considered the dimension of the embedded sets of states is considerably smaller than that of any appropriately chosen finite state space approximation and this dimensionality reduction is important in view of computations.

After having discussed the embedding approach to compute the average costs and the relative cost function, we present a number of applications. The first two applications deal with the optimal control of queues and concern an M/G/1 queue with variable service rate and an M/M/s queue in which the number of operating service channels can be varied. Next we discuss some applications in inventory and production. We first consider a production system in which discrete production occurs and the inventory can be controlled by turning on or off the

production facility. An efficient algorithm is given to compute an optimal (S,s) policy. We finally discuss a multi-item inventory system in which significant economies of scale can be exploited when coordinating replenishment orders for groups of items. For a continuous review multi-item inventory problem with compound Poisson demands and constant lead times, we present an efficient algorithm which searches for a simple coordinated control rule.

References

1. Crabill, T.B., D. Gross & M.J. Magazine (1977), A classified bibliography of research on optimal design and control of queues. Operations Research Vol. 25, 219-232.
2. Prabhu, N.U. & S. Stidham Jr. (1974), Optimal control of queueing systems, in: Mathematical Methods in Queueing Theory (A.B. Clarke ed.), 263-294, Lecture Notes in Economics and Mathematical Systems 98, Springer-Verlag, Berlin.
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Approximations of dynamic, multilocation production and inventory problems

by

Awi Federgruen
Graduate School of Business
Columbia University
New York, NY 10027

and

Paul Zipkin
Graduate School of Business
Columbia University
New York, NY 10027

ABSTRACT

Consider a central depot (or plant) which supplies several locations experiencing random demands. Orders are placed (or production is initiated) periodically by the depot. The order arrives after a fixed leadtime, and is then allocated among the several locations. The allocations are finally received at the demand points after another lag.

This system gives rise to a dynamic program with a state space of very large dimension. We show that this model can be systematically approximated by a single-location inventory problem, employing aggregation techniques. All the qualitative and quantitative results for such problems can then be applied.

KEYWORDS: INVENTORY & PRODUCTION;
APPROXIMATIONS,
STOCHASTIC MODELS

Using the problem structure in large Markov decision processes.

by

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This talk is based on the paper "Some notes on iterative optimization of structured Markov decision processes with discounted reward" by M. Hendrikx, J. van Nunen and J. Wessels (memorandum COSOR 80-20, Eindhoven University of Technology, Department of Mathematics, November 1980).

The paper contains a comparison of solution techniques for Markov decision processes with respect to the total reward criterion. It is illustrated by 4 examples (2 from literature and 2 from our own experience), that the effect of a number of improvements of iterative methods, which are advocated in the literature, is limited in some realistic situations.
Numerical evidence is provided to show that exploiting the structure of the problem under consideration often yields a more substantial reduction of the required computational effort than some of the existing acceleration procedures.
We advocate that this structure should be analyzed and used in choosing the appropriate solution procedure. This procedure might be composed by blending several of the acceleration concepts that are described in literature. The four test problems are sketched and solved with several successive approximation methods. These methods were composed after analysing the structure of the problem. The required computational efforts are compared. Among other things it appears that combination of different acceleration techniques is sometimes counterproductive. It also appears that it often is important to select the computational method on the basis of its possibilities for efficient handling, rather than on its convergence rate.

On Stationary Reliability Characteristics of Complex Systems with Repair

by

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ABSTRACT

1. In this paper a unified approach is given to derive formulas for the state probabilities, the availability and the interval reliability of a complex repairable system in steady state. The approach is based on an application of stochastic processes with an embedded point process, cf. e.g., [1]-[5].

Definition. A pair $[(X(t), -\infty < t < \infty), (T_n, -\infty < n < \infty)]$ consisting of a stochastic process $(X(t))$ with the state space $(E, \sigma(E))$ and a sequence of real valued r.v.'s (T_n) both defined on the same probability space $(\Omega, \mathscr{F}, Q)$ and satisfying the condition

$$T_n \xrightarrow[n \to \pm\infty]{} \pm\infty; \qquad \ldots < T_0 \leq 0 < T_1 < \ldots \ Q\text{-a.s.}$$

is called a process with an embedded point process (abr. PEP). The points T_n are called the embedded points.

A PEP$[(X(t), (T_n)]$ is called synchronous if the cycles $(X_n(u) \mathrel{\hat{=}} X(T_n + u)$, $0 \leq u < T_{n+1} - T_n)$, $-\infty < n < \infty$, form a stationary sequence (or variables taking values in a suitable space).

A PEP$[(\bar{X}(t)), (T_n)]$ is called stationary if for every $u \in R$ the shifted PEP $S_u[(\bar{X}(t)), (T_n)] \mathrel{\hat{=}} [(\bar{X}(t-u)), (\bar{T}_n - u)]$ has the same distribution as $[(\bar{X}(t), (\bar{T}_n)]$.

We use the notations

$\mathscr{P} = \{P$: P is the distribution of a synchronous PEP$[(X(t), (T_n)]$ with the property $0 < \Delta(P) = E_P T_1 < \infty\}$

$\bar{\mathscr{P}} = \{\bar{P}$: $\bar{P}$ is the distribution of a stationary PEP$[(X(t), (T_n)]$ with the property $0 < \lambda(\bar{P}) = E_{\bar{P}}[\#(0,1) \cap (T_n))] < \infty\}$

Theorem. There exists a one to one mapping between the families $\mathcal{P}$ and $\bar{\mathcal{P}}$. For a given $P \in \mathcal{P}$ the corresponding $\bar{P} \in \bar{\mathcal{P}}$ is determined by

$$\bar{P}([(X(t)),(\bar{T}_n)] \in A) = \Delta(P)^{-1} \int_0^\infty P(\{T_1 > u\} \cap \{S_u([(X(t)),(T_n)]) \in A\}du \quad (1)$$

From (1) we obtain the so-called stochastic mean value theorem ($f \geq 0$ and measurable)

$$E_{\bar{P}} f(\bar{X}(0)) = \Delta(P)^{-1} E_P(\int_0^{T_1} f(X(t))dt) \quad (2)$$

This formula is well-known for regenerative processes, i.e., if the cycles of a synchronous process are i.i.d., cf e.g., [6]. Formulas (1) and (2) are very useful in queueing theory, cf [5].

If the synchronous PEP$[(X(t),(T_n)]$ is metrically transitiv, one can interpret $[(X(t)),(T_n)]$ and the corresponding stationary PEP$[(\bar{X}(t)),(\bar{T}_n)]$ as two equivalent description of the same system in steady state. The first one corresponds to the point of view that the origin is an arbitrarily chosen embedded point. The second corresponds to the origin being an arbitrarily chosen point on the real axis.

2. Let us consider a complex system with repair. We assume that there exist sets G and B (G-good states, B-bad states) and that $G \cup B = E$, $G \cap B = \emptyset$. The system is failed iff its state belong to B.

As embedded points we consider the instants at which the system changes its state from B to G. If the system is in steady state we describe its temporal behavior by a synchronous PEP$[(X(t),(T_n)]$ and the corresponding (in the sense of Theorem 1) stationary PEP$[(\bar{X}(t)),(T_n)]$.

As the stationary state probabilities, stationary availability and stationary interval relability we define the quantities

$$\bar{p}_j = \bar{P}(\bar{X}(0)=j),\ A = \bar{P}(\bar{X}(0) \in G),\ A_v = \bar{P}(\bar{X}(u) \in G,\ \ 0 \leq u < v), \quad (3)$$

respectively. The lengths of generic cycle of the synchronous process can

be written in the form $\Delta = U + D$, where U is the up-time and D is the down-time of the system within the cycle. Using (1) we get

$$A_v = \frac{1}{E_P U + E_P D} \int_v^\infty P(U > t)dt \tag{4}$$

$$A = \frac{1}{E_P U + E_P D} \tag{5}$$

Both formulae are well known for the so called repairable unit if the temporal behavior of the system is described by an alternating renewal process. Formula (3) was proved in [7] for Markov systems with finite state space. (We point out that the formulas (4), (5), as well as the formulas (6) and (7) below hold for an arbitrary state space E).

3. Let $C \in \sigma(E)$ and assume that the instants in which the system enters or exits the set C belong to the sequence of embedded points. Denote

$\Delta = E_P(T_{n+1} - T_n)$, $\pi(L) = P(X(0+0) \in L)$, $L \in \sigma(E)$, $\Delta_x = E_P(T_1 | X(0+0) = x)$,

(we assume that this conditional expectation exists). Using the formula (2) for these embedded points and $f = 1_C$ we get

$$\bar{P}(\bar{X}(0) \in C) = \frac{1}{\Delta} \int_C \Delta_x \pi(dx) = \frac{\int_C \Delta_x \pi(dx)}{\int_E \Delta_x \pi(dx)} \tag{6}$$

In a reliability framework (discrete state space) we have

$$\bar{p}_k = \frac{\pi_k \Delta_k}{\sum_{j \in S} \pi_j \Delta_j}, \quad k \in E \tag{6a}$$

Consider a semi-Markov process with an arbitrary state space E such that there exists a unique invariant probability measure π of the embedded Markov chain. If we consider all jump points as embedded point the formula (6a) is the well-known formula for the stationary distribution of a semi-

Markov process, cf e.g., [1], [2]. So we can observe that the formula (5) holds in fact for arbitrary PEP. However, unlike semi-Markov processes, the determination of π and Δ_x, $x \in E$ can be quite difficult.

We get another formula for the probability $\bar{P}(\bar{X}(0) \in C)$ if we consider the entrance points into C as embedded points (there could be further embedded points, also). We denote by $((\tilde{X}(t), (\tilde{T}_n))$ and $\tilde{P}$ the synchronous PEP and its probability distribution and use the notations $\tilde{\Delta} = E_{\tilde{p}}(\tilde{T}_{n+1} - \tilde{T}_n)$, $\tilde{\pi}(L) = P(\tilde{X}(0+0) \in L)$, $L \in \sigma(E)$, $\tilde{\Delta}_x = E_{\tilde{P}}(\int_0^{T_1} 1_C(X(t))dt \,|\, X(0+0 = x)$ - the conditional mean sojourn time in C under the condition that the process starts in $x \in C$. Using (2) we get

$$P(\bar{X}(0) \in C) = \frac{1}{\tilde{\Delta}} \int_C \tilde{\Delta}_x \tilde{\pi}(dx) \tag{7}$$

For discrete E we consider $C = \{k\}$ and get

$$\bar{p}(k) = \frac{1}{\tilde{\Delta}} \tilde{\pi}(k)\tilde{\Delta}_k \tag{7a}$$

The formulas (7) and (7a) are well-known for semi-regenerative processes, cf e.g., [1], [2], [4]. In reliability theory one can use formulas (6a) and (7a) for the determination of the availability via

$$A = \sum_{k \in G} \bar{p}(k),$$

cf [1]. But our experience shows that the use of (5) leads to much simpler calculations.

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Conservation Laws for Queues

by

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ABSTRACT

Queueing systems obey a conservation law which relates certain time and customer averages. This talk will describe a generalization of Little's formula (due to S. Brumelle), give examples of its use, and sketch a new proof.

Queues with Non-Stationary Input Stream: Ross's Conjecture

by

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ABSTRACT

Characteristics of queues with non-stationary input streams are difficult to evaluate, therefore bounds for them are of importance. First we define what we understand by the stationary delay and find out stability conditions of single server queues with non-stationary inputs. For this purpose we introduce a notion of ergodically stable sequence of random variables. The worked out theory is applied to single server queues with stationary double stochastic Poisson arrivals. Then the inter-arrival times do not form a stationary sequence ("time stationary" do not imply "customer stationary"). We show that the average customer delay in the queue is greater than in a standard M/G1 queue with the same average input rate and service times. This result is used in examples which show that the assumption of stationarity of the input point process is non-essential.

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